I-DIVERGENCE FUNCTION IN PROJECT EVALUATION


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Summary

The adoption of new construction production technologies involves some degree of hazard to project developers. This is especially true when new knowledge is not tested on technical or commercial scales. In these situations project hazards are difficult to analyze. Risk analysis is a new discipline that determines the success of technical economic projects. We can divide risk analysis into two branches a) construction and design risk analysis and b) economic risk analysis. Development in construction and technology carries new hazards, which endanger construction projects. In economic risk analysis, the entire project on a wide risk scale is analyzed along with its influence on efficiency and commercial attractiveness. In these analyses, it is essential to use time value calculations because all hazards are usually classified through their time dependent influence on the costs and benefits of projects.

Keywords: Divergence, divergence function, risk analysis, project evaluation, economic analysis

1 Introduction

The decision-making in its technical design is an important step in the attainment of technical rationality and economic transparency. However we are able to set up the main framework for solutions as the maximum rational investment volume

$$I_{max} = NP \bullet t_{eco} \quad (1a),$$

where NP is *net profit* and is given as a hypothetical profit per year and $t_{eco} = 1/r$, where r is the **expected interest rate**. This pronouncement might be easy if I_{max} was really constant and not $I_{max}(t, \ldots)$, where three dots is an easy way to deal with life cycle cost (LCC) parameters, quality, architectural attractiveness, location of investment in any decision focus. However the I_{max} gives us a frame for more sophisticated methodological approaches, that are able to solve such a complicated scheme presented here as dots. The evaluation of economic data in long time perspective is difficult and **long-term goods** (housing) and **very long-term goods** (urban structure, roads, infrastructure) need evaluation. There might be useful a rough evaluation on the basis of the search for *t* with *max* difference in (1b)

$$max \sum_{t} \; [NP(t) - I_{\max}(t) - LCC(t)] \text{ for } t=1, 2, \ldots, \text{LC} \qquad (1b).$$

We will present this as $t_{\max}$ and its value will indicate the rational sustainable life cycle of the building construction in substance. In expression (1) we will simplify the situation and discover the possibility of evaluation of difference in variants given by costs $N(t)$.

2 An example of the assessment of maintenance costs

For a graphic assessment of maintenance costs restoration there was chosen an illustrative example of a brick laid apartment house construction object (Holubičková, Nováková, 2007). The construction is carried out by a classical technology with five aboveground floor levels. The construction object is being solved without a basement. The basic descriptive data gives the following **Tab. 1**. When assessing costs the situation was for simplicity reduced to constantly spent amounts in definite time periods. It is supposed that percentage maintenance costs will in a more distant time horizon be subject to increase (see the assignment in **Tab. 2**).

Tab. 1 The basic object information

Built-up area (m^2)	49275
Utility area (m^2)	16461
Enclosure area (m^3)	3113
Technologies	blockwork
Price	14 881 000 CZK

Tab. 2 Percentage maintenance costs (var. 1)

Coefficient of deterioration (Var. 1) … (%)			
% (1-20 let)	% (21-40 let)	% (41-60 let)	% (61-80 let)
0,6	1,1	1,5	2,1

Tab. 3 Percentage maintenance costs (var. 2)

Coefficient of deterioration (Var. 2) … (%)			
% (1-20 let)	% (21-40 let)	% (41-60let)	% (61-80 let)
0,66	0,7	1,8	1,9

The postulating of percentage ratios is chosen as an illustrative assignment[1]. For any practical solution there would be necessary a more thorough analysis of a project involving a more detailed subdivision of time scales and individual construction elements in the

[1] Variation in the assignment can occur from the viewpoint of a different approach, for example of the manager of the object, towards the total concept of maintenance.

structural object (more details in Čápová et al. 2005, Prostějovská, 2006). For the needs of this task assessment such an assignment is (see. **Tab. 1**, **2**) sufficient. There is schematically typical as a structural object a floor with three flats on a landing depicted in **Fig. 1**. The resulting cost values on structural objects maintenance for both variants are given in **Fig. 2** and result from the assignment of the entry values given in **Tab. 2** and **3**.

Fig. 1 Scheme of living house

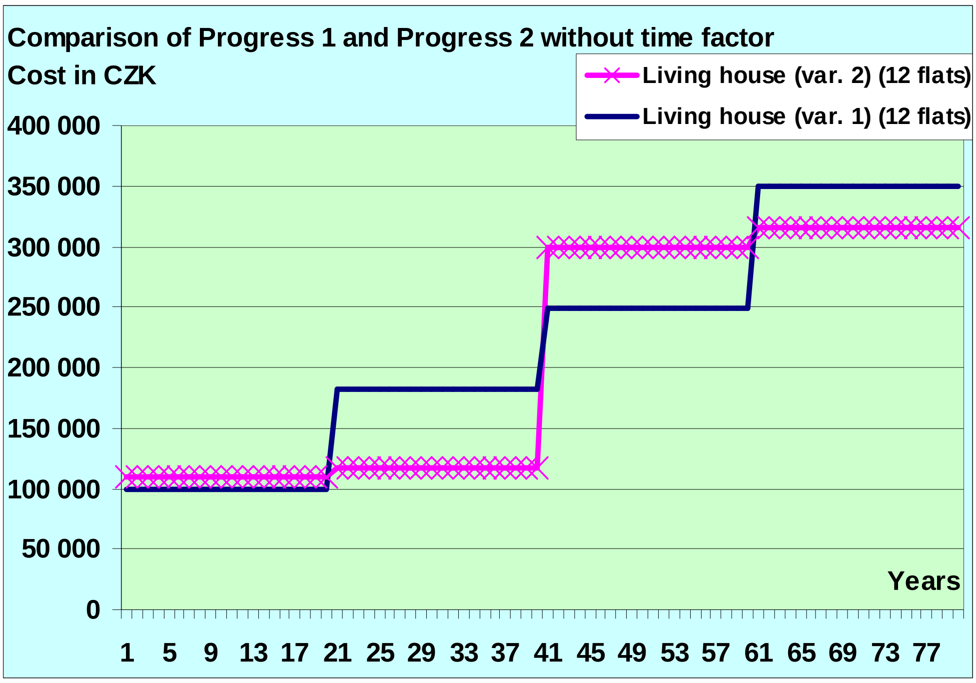

Fig. 2 Two variants of cost progression without time factor

It is necessary to keep these data as entries, so that they can be subject to a wider professional discussion on their evaluation (detailed in Beran et al., 2006). In the opposite case of a constant assignment the general usability of the presented method is significantly limited.

In **Fig. 2** there are given cost progressions for a monitored period of 80 years without considering the time factor. The different understanding of the structural object maintenance comes from the assignments in **Tab. 2** and **3**. The following **Fig. 3** depicts the maintenance costs with the time factor consideration. It is possible to monitor the different strategies in chosen time stages (20, 40, 60, 80 years), at which there is arrived at the varying excess of maintenance costs. Typically this situation is obvious from a symbolic notation of costs

$$N^{Var.1}_{(40\,\text{až}\,60\,\text{let})} < N^{Var.2}_{(40\,\text{až}\,60\text{let})} \tag{1}$$

$$N^{Var.1}_{(60\,\text{až}\,80\,\text{let})} > N^{Var.2}_{(60\,\text{až}\,80\text{let})} \tag{2}$$

where N represents maintenance costs in defined time intervals (40 up to 60 years and 60 up to 80 years) for defined variants (Var. 1 and Var. 2).

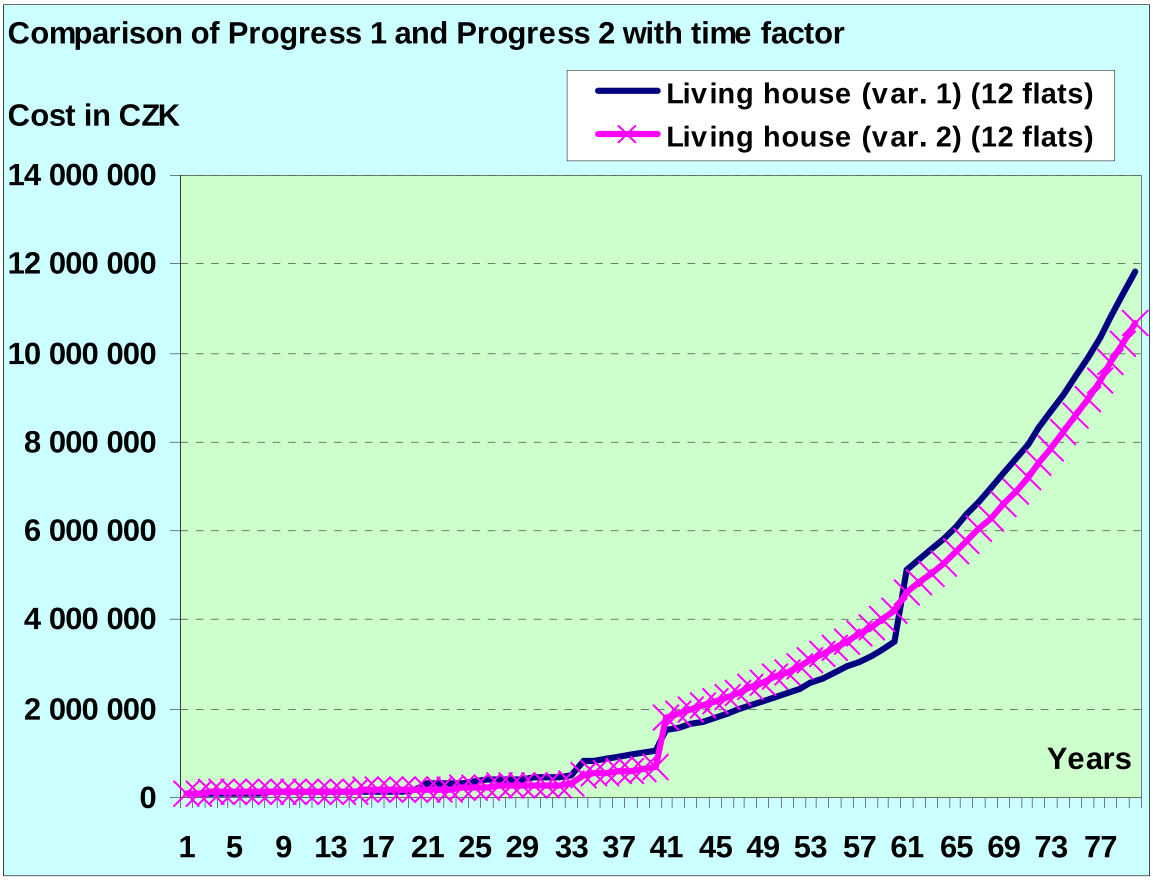

Fig. 3 Two variants of cost progression with time factor

3 Theory of the divergence function

The set of decision factors **F** can be both one-dimensional (concerning one decision factor) and multi-dimensional (concerning *n* decision factors). To create a difference in the area of one-dimensional evaluations of two variants for example *A Z* means finding the distance between the evaluation $d(h^A;h^Z)$. Similarly a two-dimensional evaluation requires finding $d(h^A{}_1,h^A{}_2;h^Z{}_1,h_2{}^Z)$. We will use and modify a form of notation taken from Czisar (1991). We will mark the divergence between the evaluation of variants *A* and *Z* as $D(A\|Z)$ possibly $div(A\|Z)$, where $\boldsymbol{h}^A = (h^A{}_1, h^A{}_2,\ldots, h^A{}_j,\ldots,h^A{}_n)>0$ and $\boldsymbol{h}^Z = (h^Z{}_1, h^Z{}_2,\ldots, h^Z{}_j,\ldots,h^Z{}_n)>0$ are vectors of individual evaluations. The calculation of the divergence as proposed by Czisar (1991), Beran (2006) is

$$D(A\|Z) = \sum_{j=1}^{n} (h^A{}_j \ln(h^A{}_j/\, h^Z{}_j) - h^A{}_j + h^Z{}_j) \tag{3}$$

for all $j = 1,2,\ldots,n$ of decision factors of the set F. This general notation can be symbolically expressed for vectors

$$a = (a_1, a_2, ..., a_n) \tag{4}$$

$$b = (b_1, b_2, ..., b_n) \tag{5}$$

where $n \in X_N$. When fixing the terms of their divergence with the help of the I-divergence relationship there is then validated

$$D(a \,\|\, b) = \sum_{i=1}^{n} a_i \log\left(\frac{a_i}{b_i}\right) \tag{6}$$

or its modified alternative

$$D(a \,\|\, b) = \sum_{i=1}^{n} \left(a_i \ln\left(\frac{a_i}{b_i}\right) - a_i + b_i \right) \tag{7}$$

When applying the procedures there is then rendered valid that

$$D(a \,\|\, b) \neq D(b \,\|\, a) \tag{8}$$

The function (3) we will call the function **I-divergence**. If the evaluation of the variant $\boldsymbol{h}^Z$ is the starting point evaluation (given, referential, standard, normative and suchlike)[2] solution $\boldsymbol{h}^A$, $\boldsymbol{h}^B$, $\boldsymbol{h}^C$,… presents a variable solution. Variants *A* and *Z* are then *identical* on the basis of the function **I-divergence** only if all elements of the evaluation are equal $\boldsymbol{h}^A$ and $\boldsymbol{h}^Z$. Two variants *A* and *Z* are on the basis of the **I-divergence function** (2) **equivalent** when $D(A\|Z) = 0$.

The following **Tab. 4** clearly gives the calculation of divergence progression for individual variants including their mutual comparison. The second and the third column represent calculation costs for the maintenance of variants 1 and 2. In the fourth column there is assigned a referential variant to which the other variants are compared. The last two columns are the resulting values of the divergence functions for var. 1 and 2.

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[2] The referential variant can be considered for example as a maximum, a minimum or as a result established to a certain level of reliability.

Tab. 4 Calculation of divergence

Year	Variants		Refer. vector	Divergence calculation	
	Vector *A* (var. 1)	Vector *B* (var. 2)	*C* (possible)	Vector *D*(*A*‖*C*)	Vector *D*(*B*‖*C*)
1	101784	111962	*2000000*	1595098,3	1565279,3
2	103820	114202	*2000000*	1589056,2	1558846,4
3	105896	116486	*2000000*	1582934,3	1552330,0
4	108014	118815	*2000000*	1576732,0	1545729,5
5	110174	121192	*2000000*	1570448,3	1539044,0
6	112378	123615	*2000000*	1564082,7	1532272,8
7	114625	126088	*2000000*	1557634,2	1525415,1
8	116918	128610	*2000000*	1551102,2	1518470,2
9	119256	131182	*2000000*	1544485,8	1511437,3
10	121 641	133 805	*2000000*	1537784,3	1504315,8
11	124 074	136 481	*2000000*	1530997,0	1497104,8
12	134 204	147 624	*2000000*	1503238,2	1467632,2
13	137 559	151 315	*2000000*	1494215,9	1458059,3
75	9 481 325	8 578 342	*2000000*	7273293,8	5912525,2
76	9 907 985	8 964 367	*2000000*	7946710,6	6483172,4
77	10 353 844	9 367 763	*2000000*	8670056,6	7097254,9
78	10 819 767	9 789 313	*2000000*	9446461,6	7757526,5
79	11 306 656	10 229 832	*2000000*	10279236,1	8466900,5
80	11 815 456	10 690 174	*2000000*	11171881,3	9228459,2
				155340855,8	142204244,8
				D(*A*‖*C*)	**D(*B*‖*C*)**

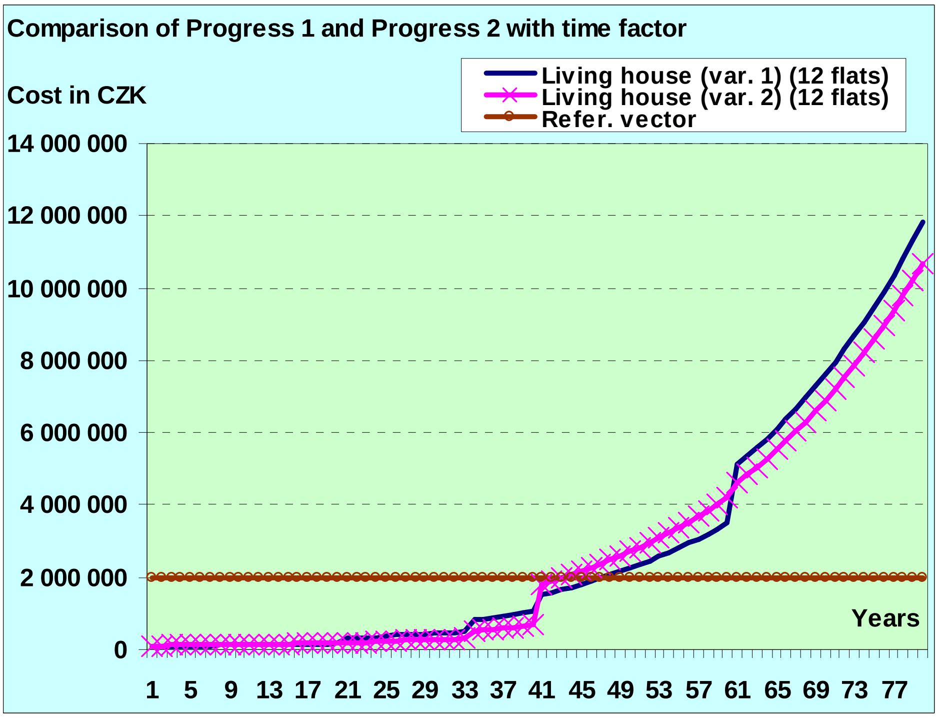

Fig. 4 Two variants of cost progression with reference vector

In **Fig. 4** it is seen that for a mutual comparison of Var. 1 and Var. 2 there was chosen a referential variant on a level of maintenance costs 2 000 000 CZK. Both variants are then evaluated in relation to the I-divergence function.

Calculated divergence progression among the assigned variants and the referential vector are given in **Fig. 5**. From the definition of a divergence relation there results that if the compared value is the same as the referential one, the resulting divergence is zero. The more the values differ, the higher they go in divergence values. This quality is clearly obvious around the 50 year time period.

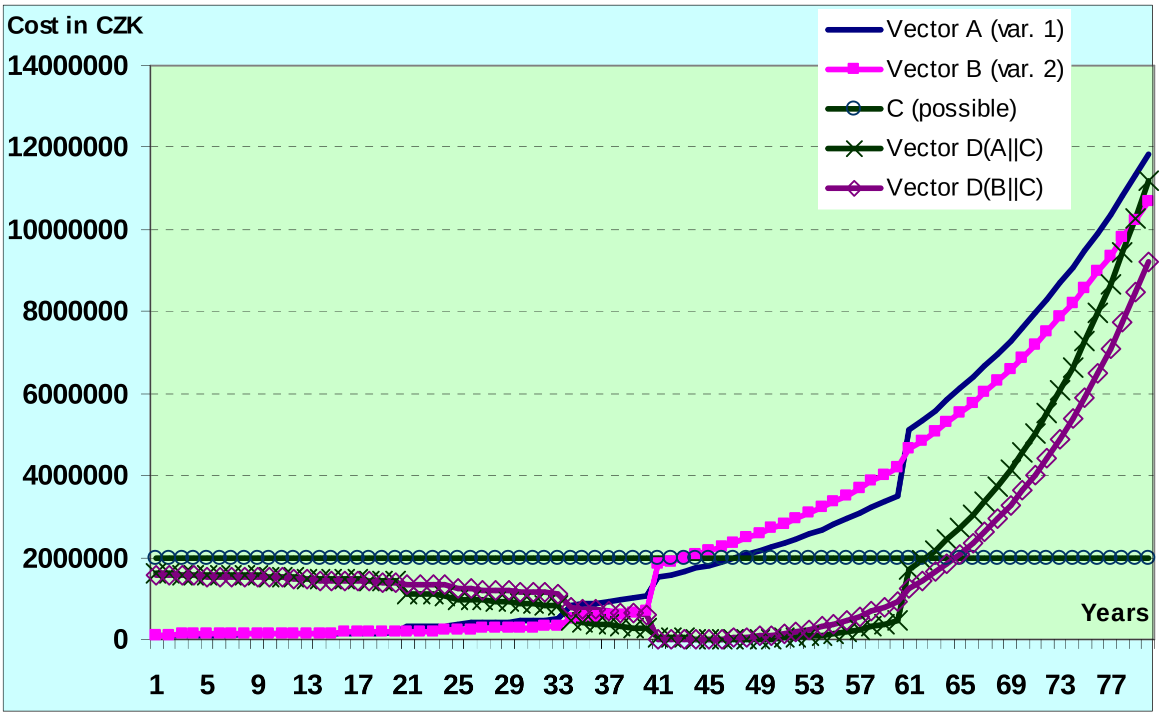

Fig. 5 Cost progression and progression of divergence vector

In the initial phases both variants are distant from the referential variant. With the passing of time also the maintenance costs grow and approach the referential variant. In this time phase the divergences approach the zero level. In the final phase of monitoring the development of Var. 1 and Var. 2 we come to the separating out of the costs from the referential variant and at the same time thus to the raising of the divergence vectors ($D(A||C)$, $D(B||C)$).

4 Conclusions

Practical cases of using construction objects and their maintenance at the present time show that in many cases there does not exist a conceptual approach to this complex of problems. The result is then the subsequently failing functions of structural objects, which lower the original standard assigned by a project. In a worse case the use of the construction object is even thwarted or leads in limited cases to their total demolition. In the proposed regime of the management it is necessary to decide whether it is functional and feasible in future.

The given illustrative example in schematic view offers the possibility of the evaluation of more viewpoints on this complex of problems. The final choice should be a rational view on the solution of this problem, using for any decision the example given by the theory of I-divergence functions.

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